

Exercise 73

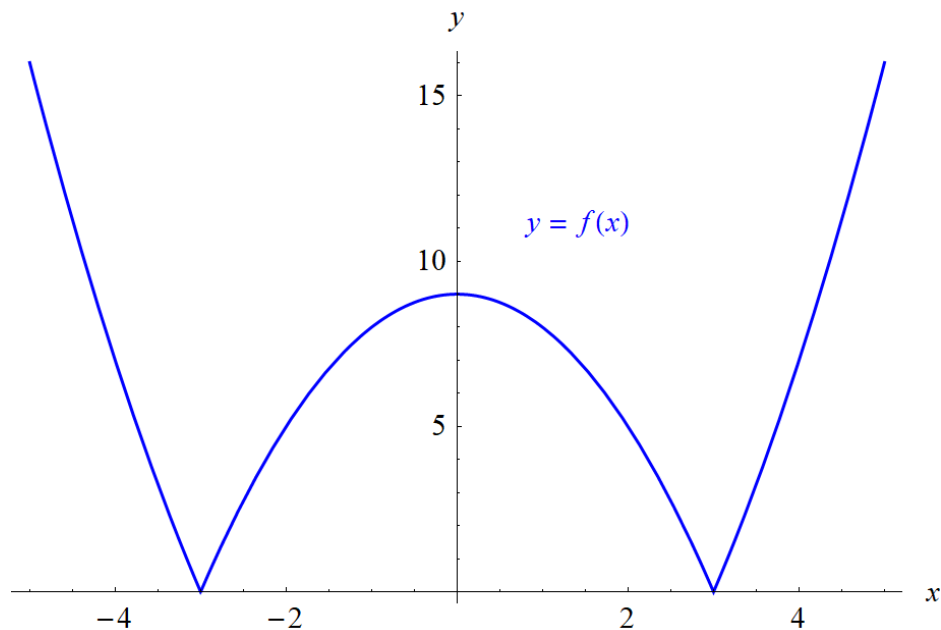
- (a) For what values of x is the function $f(x) = |x^2 - 9|$ differentiable? Find a formula for f' .
- (b) Sketch the graphs of f and f' .

Solution

Rewrite the function for $f(x)$.

$$f(x) = |x^2 - 9| = \begin{cases} x^2 - 9 & \text{if } x^2 - 9 \geq 0 \\ -(x^2 - 9) & \text{if } x^2 - 9 < 0 \end{cases} = \begin{cases} x^2 - 9 & \text{if } x \leq -3 \\ x^2 - 9 & \text{if } x \geq 3 \\ 9 - x^2 & \text{if } -3 < x < 3 \end{cases}$$

Below is a graph of $f(x)$ versus x .



Although the function is continuous, there are kinks in the curve at $x = \pm 3$, which means the slope (or derivative) is undefined there. That is, f is not differentiable at ± 3 . The derivative of f is

$$f'(x) = \begin{cases} 2x & \text{if } x \leq -3 \\ 2x & \text{if } x \geq 3 \\ -2x & \text{if } -3 < x < 3 \end{cases},$$

and its graph versus x is shown below.

